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COMMENTS ON "NON-LINEAR VIBRATIONS OF A BEAM–MASS SYSTEM UNDER DIFFERENT BOUNDARY CONDITIONS"

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Large amplitude free flexural vibrations of slender beams have been investigated by several researchers [1–4]. The effect of axial displacement is considered by Raju *et al*. [1], in which the strain energy U and the kinetic energy T of the unloaded beam are given by

$$U = \int_0^l \left[EA\left(\frac{\partial u}{\partial x} + \tfrac{1}{2}\left\{\frac{\partial w}{\partial x}\right\}^2\right)^2 + EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \right] \mathrm{d}x,$$

$$T = \tfrac{1}{2}\rho A\omega^2 \int_0^l (u^2 + w^2)\,\mathrm{d}x,$$

where u is the axial displacement due to a tensile force. Raju *et al*. [1] concluded that the effect of the longitudinal deformation and inertia is to reduce the non-linearity, and the longitudinal inertia is negligible for slender beams. In reference [3], a perturbation method is used to analyze the non-linear vibrations of a beam–mass system under different boundary conditions. Its final model ignores the axial displacement, but includes the viscous damping and the external excitation [3].

In a recent note [5], different assumed shape functions are used, one at a time, to obtain the kinetic and potential energies of the three classical beams carrying a concentrated mass at various positions. A closed-form expression for the fundamental frequency of each case is then written in terms of mass ratio and position parameter [5] as,

$$\omega^2 = [EI/(Ml^3)]\,[K(A_\alpha + A)/(B_\alpha + B)]. \tag{1}$$

In reference [3], the transcendental frequency equations are given for different end conditions. The effects of the position and magnitude of the mass on the first five frequencies are investigated [3].

For comparison of these models, those equations for the simply-supported beam carrying concentrated mass are given as follows [3, 5]:

$$y_W = \frac{Wbx}{6lEI}(l^2 - x^2 - b^2), \qquad \text{for } 0 \leqslant x \leqslant a; \tag{2a}$$

$$y_W = \frac{Wa(l-x)}{6lEI}(2lx - x^2 - a^2), \qquad \text{for } a \leqslant x \leqslant l; \tag{2b}$$

$$y_m = \frac{wx}{24EI}(x^3 - 2lx^2 + l^3), \qquad \text{for } 0 \leqslant x \leqslant l; \tag{3}$$

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$$y_c = y_m + y_W, \qquad \text{for } 0 \leqslant x \leqslant l; \tag{4}$$

$$y_t = C \sin \frac{\pi x}{l}, \qquad \text{for } 0 \leqslant x \leqslant l; \tag{5}$$

$$2 \tanh \beta \tan \beta + \alpha\beta\{\tanh \beta \sin \beta\eta(\sin \beta\eta - \tan \beta \cos \beta\eta) + \tan \beta \sinh \beta\eta(\tanh \beta \cosh \beta\eta - \sinh \beta\eta)\} = 0. \tag{6}$$

Note that equations (2–5) are the respective shape functions that will each be used in Rayleigh's quotient for the frequency analysis [5], while equation (6) is given in reference [3] for multi-mode frequencies associated with different mass ratios ($\alpha = M/(\rho A l)$) and locations ($\eta = x/l$).

Rewrite equation (1) as $\omega^2 = \bar{\omega}^2 EI/(ml^3)$, where $\bar{\omega} = \beta^2$. Table 1 compares the fundamental frequencies obtained via equations (2–6). Several points are worth noting from Table 1.

1. The frequencies obtained from the shape function y_c (equation (4)), which involves both the distributed beam mass m $(= wl/g)$ and the weight's mass M $(= W/g)$, are closer to those evaluated by the transcendental equation (6).
2. As expected, the frequencies are unchanged with respect to different weights at the ends ($\eta = 0$ and 1) due to the zero displacement.
3. All models give similar results for cases with the weight placed near the beam's centre. The same finding is concluded in reference [6].
4. In general, the frequencies obtained by using y_m and y_t are higher than others.
5. As stated in reference [7], the model generated by using y_W (weight only) must not be used if the weight is placed near the beam's ends as the frequencies obtained are quite high and inaccurate.

It is concluded that the model using y_c can be used to *quickly* obtain the fundamental frequency of loaded beams, owing to its simple algebraic expression (see reference [5] for the full expression). Nevertheless, the transcendental expression (6) is useful for cases of higher-mode frequencies.

In references [2] and [4], both experimental and theoretical results were presented for beams carrying a concentrated mass at mid-span. By virtue of the Rayleigh–Ritz procedure and by adding a tensile force, a multi-term series with sine function was used for deflection

TABLE 1

Fundamental frequency of loaded beams for different mass ratios and locations

α	η	$\bar{\omega}$ (eq. (6))	$\bar{\omega}$ (eq. (2))	$\bar{\omega}$ (eq. (3))	$\bar{\omega}$ (eq. (4))	$\bar{\omega}$ (eq. (5))
1	0·0	9·8695	12·5499	9·8767	9·8767	9·8696
	0·1	8·9962	9·86805	9·0328	9·0432	9·0437
	0·2	7·4541	7·63423	7·5749	7·4575	7·5898
	0·3	6·3946	6·43678	6·4953	6·3958	6·4951
	0·4	5·8468	5·85753	5·9026	5·8482	5·8887
	0·5	5·6795	5·68399	5·7170	5·6809	5·6982
10	0·0	9·8695	12·5499	9·8767	9·8767	9·8696
	0·1	5·3322	5·37843	5·7448	5·3409	5·7858
	0·2	3·2598	3·26237	3·4918	3·2599	3·5093
	0·3	2·5279	2·52832	2·6283	2·5279	2·6293
	0·4	2·2252	2·22527	2·2659	2·2252	2·2589
	0·5	2·1395	2·13955	2·1632	2·1395	2·1537

curves for modes higher than the first [2]. A further study by Low *et al*. [4] suggests a better model for the frequency of non-linear beams carrying a concentrated mass at $x = a$:

$$\omega^2 = \left[\int EI\left(\frac{d^2y}{dx^2}\right)^2 dx + \int \frac{EA}{4}\left(\frac{dy}{dx}\right)^4 dx\right] \Bigg/ \left[\int \rho A y^2\, dx + My^2|_{x=a}\right], \qquad (7)$$

in which the term of bending slope (dy/dx) is also considered in reference [3]. Its axial effect will act to increase the frequency.

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AUTHORS' REPLY

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We would like to thank Prof. K. H. Low [1] for his comments on our paper [2] and detailed comparisons between our paper and some of the relevant work. While we agree with his comments and conclusions, to clarify the issue, two minor comments are listed below:

1) For the model presented in our paper [2] the comment that "Its final model ignores the axial displacement" is not precise. A more detailed derivation of our model may be found in [3]. In that paper the axial displacement is taken into account and then eliminated between the coupled equations by using integration.

2) It is worth noting that our transcendental equation as well as the mode shapes were derived based on exact methods. It might be expected that the solutions based on approximate methods would converge to those of ours when better mode shapes are chosen.

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